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Coupled-Transmission-Line Directional Couplers with Coupled Lines of Unequal Characteristic Impedances

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Abstract—A new class of coupled-transmission-line directional couplers, called "nonsymmetrical directional couplers," is described. Unlike conventional directional couplers, nonsymmetrical directional couplers use coupled lines of unequal characteristic impedances. The principal difference between the performance of nonsymmetrical directional couplers and that of conventional designs is the impedance level of the coupled waves, which may be changed to higher or lower impedance levels than that of the incident wave. These directional couplers may be designed to have infinite directivity and to be matched at all frequencies, or they may be designed to have infinite directivity at all frequencies and a specified maximum VSWR. Coupling relationships and design equations for both cases are presented, and the relative properties of both cases are discussed. The theoretical limitation on the maximum coupling and the maximum impedance transformation that can be obtained simultaneously are derived. Techniques for broadbanding by cascading additional sections of coupled lines are described. Experimental results of a trial −10-dB coupler with coupled lines of 50 and 75 ohms are presented.

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Introduction

In the past, coupled-transmission-line directional couplers have been designed with coupled lines of equal characteristic impedances [1]–[6]. These couplers are used in many applications: power samplers, reflectometers, directional detectors, directional filters, and multiplexers are several. In this paper a new class of coupled-transmission-line directional couplers, called nonsymmetrical directional couplers, is described. In contrast to conventional directional couplers, nonsymmetrical directional couplers use coupled lines of unequal characteristic impedances. The principal difference between the performance of nonsymmetrical directional couplers and that of conventional directional couplers is the impedance level of the coupled waves, which may be changed to higher or lower impedance levels than that of the incident wave. Nonsymmetrical directional couplers, therefore, act as conventional directional couplers combined with impedance transformers at the two ports of one of the transmission lines. Thus, nonsymmetrical directional couplers should prove useful in applications calling for directional coupling and impedance transforming in combination, since these functions can be accomplished in a single device. For example, a practical application for which the new device may prove useful is in directional detectors [7], wherein the diode detector might be better matched in a line of different impedance from that of the main line.

The nomenclature *nonsymmetrical* directional coupler pertains to the side-by-side asymmetry of the directional coupler. It should not be confused with *cascaded* asymmetrical [4] or *cascaded* symmetrical [5], [6] directional couplers, which use coupled lines of equal characteristic impedances and have end-to-end asymmetry or symmetry, respectively.

Figure 1 shows the nonsymmetrical coupled-transmission-line directional coupler in diagrammatical form, and it also specifies voltage and current coordinates used throughout this paper for such couplers. The structure in the figure is to be regarded as two uniformly coupled transmission lines of unequal characteristic impedances that have equal propagation constants in balanced (odd-mode) and unbalanced (even-mode) excitation [1]. Although the structure shown has end-to-end geometrical symmetry, the conclusions and design equations presented herein only require that the structure have end-to-end electrical symmetry. While nonsymmetrical directional couplers may be realized in many configurations that satisfy these conditions, the electrical characteristics are independent of the physical configuration. The admittance matrix corresponding to the voltage and current coordinates of Fig. 1 is [8][1]

$$\begin{bmatrix} \frac{Y_{oo}^a+Y_{oe}^a}{2}\cdot\frac{1}{s} & -\frac{Y_{oo}^a-Y_{oe}^a}{2}\cdot\frac{1}{s} & \frac{Y_{oo}^a-Y_{oe}^a}{2}\cdot\frac{\sqrt{1-s^2}}{s} & -\frac{Y_{oo}^a+Y_{oe}^a}{2}\cdot\frac{\sqrt{1-s^2}}{s} \\ -\frac{Y_{oo}^b-Y_{oe}^b}{2}\cdot\frac{1}{s} & \frac{Y_{oo}^b+Y_{oe}^b}{2}\cdot\frac{1}{s} & -\frac{Y_{oo}^b+Y_{oe}^b}{2}\cdot\frac{\sqrt{1-s^2}}{s} & \frac{Y_{oo}^b-Y_{oe}^b}{2}\cdot\frac{\sqrt{1-s^2}}{s} \\ \frac{Y_{oo}^b-Y_{oe}^b}{2}\cdot\frac{\sqrt{1-s^2}}{s} & -\frac{Y_{oo}^b+Y_{oe}^b}{2}\cdot\frac{\sqrt{1-s^2}}{s} & \frac{Y_{oo}^b+Y_{oe}^b}{2}\cdot\frac{1}{s} & -\frac{Y_{oo}^b-Y_{oe}^b}{2}\cdot\frac{1}{s} \\ -\frac{Y_{oo}^a+Y_{oe}^a}{2}\cdot\frac{\sqrt{1-s^2}}{s} & \frac{Y_{oo}^a-Y_{oe}^a}{2}\cdot\frac{\sqrt{1-s^2}}{s} & -\frac{Y_{oo}^a-Y_{oe}^a}{2}\cdot\frac{1}{s} & \frac{Y_{oo}^a+Y_{oe}^a}{2}\cdot\frac{1}{s} \end{bmatrix} \quad (1)$$

where $s=\tanh \gamma L$, γ being the complex propagation constant and L the length of the coupled section. In the important case of lossless lines, $s=j \tan \theta$, where θ is the electrical length of the coupled sections, and $j=\sqrt{-1}$.

Y_{oe}^a is the admittance measured at Port 1 for an infinite section of coupled lines excited in the even mode with voltage sources at Ports 1 and 2.

Y_{oo}^a is the admittance measured at Port 1 for an infinite section of coupled lines excited in the odd mode with voltage sources at Ports 1 and 2.

Y_{oe}^b is the admittance measured at Port 2 for an infinite section of coupled lines excited in the even mode with voltage sources at Ports 1 and 2.

Y_{oo}^b is the admittance measured at Port 2 for an infinite section of coupled lines excited in the odd mode with voltage sources at Ports 1 and 2.

The following notation simplifies the design equations presented later and is therefore adopted:

$$A = \frac{Y_{oo}^a + Y_{oe}^a}{2} \quad (2)$$

$$B = \frac{Y_{oo}^b + Y_{oe}^b}{2} \quad (3)$$

$$D = \frac{Y_{oo}^a - Y_{oe}^a}{2} = \frac{Y_{oo}^b - Y_{oe}^b}{2}. \quad (4)$$

G_a and G_b are the terminating admittances of lines a and b, respectively.

In the derivations and equations that follow, excitation at Port 1 has been assumed. However, had excitation at Port 2 been assumed, the results could be made

[1] B. M. Schiffman pointed out that the Y-matrix given in Ozaki and Ishii [8] is incorrect with respect to the signs of its entries; the correct matrix is given by (1).

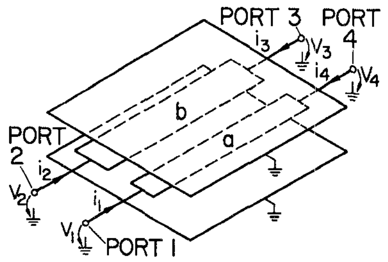

Fig. 1. Nonsymmetrical coupled-transmission-line directional coupler with coupled lines of unequal characteristic impedances.

to carry over directly by an interchange of G_a and G_b and an interchange of A and B. All formulas presented in the following sections were obtained by operating on the admittance matrix of (1) directly. The resulting formulas are thus exact for all values of coupling.

Condition for Infinite Directivity

The directivity of a directional coupler may be defined as follows. Let each transmission line of the coupler be terminated in its respective load admittance at Ports 2, 3, and 4. Let a signal be incident at Port 1. The directivity in decibels is given by

$$20 \log_{10} \left| \frac{v_2}{v_3} \right|. \tag{5}$$

Thus the condition for infinite directivity is that v_3 be equal to zero.

A method of solving for the directivity is given in Appendix I. In the interest of shortening the presentation, only the final result is given here. The condition for infinite directivity is found to be[2]

$$G_a G_b = AB - D^2. \tag{6}$$

There is no other constraint. Note that G_a and G_b need not be real to satisfy (6), although in the remainder of this section they are so assumed.

Conditions for Impedance Matching

A method of determining the conditions for impedance matching is given in Appendix II; again, only the final results are presented here. There are two conditions for impedance matching. They are

$$\frac{G_a}{G_b} = \frac{A}{B} \tag{7}$$

$$G_a G_b = AB - D^2. \tag{8}$$

Notice that the condition for infinite directivity is included in the conditions for impedance matching.

Thus a matched directional coupler must have infinite directivity. The converse, however, need not be so. A nonsymmetrical directional coupler may have infinite directivity without being matched. (This conclusion may also be established using the scattering matrix for a directional coupler [9].)

Additional consequences of these conditions are described later in the section in which design equations for mismatched directional couplers having infinite directivity are presented.

Equations (7) and (8) may be solved for G_a and G_b, giving

$$G_a = \sqrt{\frac{A}{B}} \sqrt{AB - D^2} \tag{9}$$

$$G_b = \sqrt{\frac{B}{A}} \sqrt{AB - D^2}. \tag{10}$$

Note that the right sides of (9) and (10) are real positive numbers for all physically realizable values of A, B, and D. Therefore, the right side of (9) and (10) may be interpreted as the characteristic admittances of lines a and b in the presence of each other.

Voltage and Power Coupling Relationships

By procedures similar to those discussed in Appendixes I and II, the transfer ratio v_2/i_1 in the lossless case is found to be

$$\frac{v_2}{i_1} = \frac{jk \tan \theta}{\sqrt{1 - k^2} + j \tan \theta} \frac{1}{\sqrt{G_a G_b}}, \tag{11}$$

and the coupled power ratio is found to be

$$\frac{P_2}{P_a} = \frac{k^2 \tan^2 \theta}{(1 - k^2) + \tan^2 \theta}, \tag{12}$$

where

$$k = \frac{D}{\sqrt{AB}} \tag{13}$$

is the square root of the power coupling coefficient, and P_a is the available power at Port 1 from a current source of strength I with internal conductance G_a.

Similarly, it is found that

$$\frac{v_4}{i_1} = \frac{1}{G_a} \frac{\sqrt{1 - k^2}\sqrt{1 + \tan^2 \theta}}{\sqrt{1 - k^2} + j \tan \theta}, \tag{14}$$

$$\frac{P_4}{P_a} = \frac{(1 - k^2)(1 + \tan^2 \theta)}{(1 - k^2) + \tan^2 \theta}. \tag{15}$$

From (11) and (14) it is seen that v_2 and v_4 are 90 degrees out of phase at all frequencies. Comparisons of (12) and (15) with the results given in Jones and Bolljahn [1] show that the coupled power and bandwidth of nonsymmetrical directional couplers are identical to those of conventional directional couplers, provided that the coefficients of coupling in the two cases are the same.

[2] Using a low frequency approximation, Firestone [9] showed that the condition for infinite directivity for coupled open-wire lines is (in our notation) $G_a G_b = C_m/L_m$, where C_m and L_m are the mutual capacitance and inductance per unit length of the coupled lines. It can be shown that this equation and (6) are equivalent, so that in fact the expression originally derived by Firestone for electrically short couplers is true in general.

Design Equations

Recall that the coefficient of coupling and the characteristic admittance of the coupled lines are given by

$$k = D/\sqrt{AB},$$

$$G_a = \sqrt{\frac{A}{B}}\sqrt{AB - D^2},$$

$$G_b = \sqrt{\frac{B}{A}}\sqrt{AB - D^2}. \qquad (16)$$

Equation (16) may be solved in conjunction with (2)–(4) to yield

$$Y_{oe}{}^a = \frac{G_a - k\sqrt{G_aG_b}}{\sqrt{1 - k^2}} \qquad (17)$$

$$Y_{oo}{}^a = \frac{G_a + k\sqrt{G_aG_b}}{\sqrt{1 - k^2}} \qquad (18)$$

$$Y_{oe}{}^b = \frac{G_b - k\sqrt{G_aG_b}}{\sqrt{1 - k^2}} \qquad (19)$$

$$Y_{oo}{}^b = \frac{G_b + k\sqrt{G_aG_b}}{\sqrt{1 - k^2}}\,. \qquad (20)$$

Equations (17)–(20) are the fundamental equations from which nonsymmetrical (or conventional) directional couplers may be designed from specifications that give the coupling and terminating admittances.

The theoretical limitation on the amount of impedance transformation and coupling that can be accomplished simultaneously is readily derived from (17) and (19). In order for $Y_{oe}{}^a$ and $Y_{oe}{}^b$ to be positive, it is necessary that

$$\frac{1}{k^2} \geq \frac{G_a}{G_b} \quad \text{and} \quad \frac{G_b}{G_a}\,. \qquad (21)$$

Equation (21) may be stated in words: *Let the impedance (or admittance) transformation ratio of the directional coupler be taken as larger than 1. Then, the reciprocal of the power coupling coefficient must be greater than or equal to the impedance (or admittance) transformation ratio.*

Thus, for example, a single-section −3.01-dB coupler has a theoretical limit of impedance transformation ratio of 2 to 1; a single-section −10-dB coupler has a theoretical limit of impedance transformation ratio of 10 to 1.

Figure 2 shows in diagrammatical form a general type of geometry frequently used in constructing shielded coupled-transmission-line directional couplers. Also shown are the several capacitances associated with the coupled lines when operated in the TEM mode.

The capacitance matrix associated with Fig. 2 is

$$C = \begin{pmatrix} C_a + C_{ab} & -C_{ab} \\ -C_{ab} & C_b + C_{ab} \end{pmatrix}. \qquad (22)$$

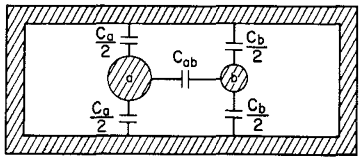

Fig. 2. A type of geometry frequently used in constructing shielded coupled-transmission-line directional couplers. (The lines a and b may be of any useful geometrical form.)

It is evident from Fig. 2 and (22) that the electrical characteristics of a coupled-transmission-line directional coupler of the type shown in Fig. 2 are completely specified when the capacitances C_a, C_b, and C_{ab} are known. The capacitances are related to the even- and odd-mode admittances by the following relationships:

$$Y_{oe}{}^a = vC_a \qquad (23)$$

$$Y_{oe}{}^b = vC_b \qquad (24)$$

$$Y_{oo}{}^a = v(C_a + 2C_{ab}) \qquad (25)$$

$$Y_{oo}{}^b = v(C_b + 2C_{ab}), \qquad (26)$$

where v is the velocity of propagation in the medium in which the waves travel.

Equations (17)–(20) may be solved in conjunction with (23)–(26) to yield the following convenient dimensionless expressions:

$$\frac{C_a}{\epsilon} = \frac{376.7}{\sqrt{\epsilon_r}}\left\{\frac{G_a - k\sqrt{G_oG_b}}{\sqrt{1 - k^2}}\right\} \qquad (27)$$

$$\frac{C_b}{\epsilon} = \frac{376.7}{\sqrt{\epsilon_r}}\left\{\frac{G_b - k\sqrt{G_oG_b}}{\sqrt{1 - k^2}}\right\} \qquad (28)$$

$$\frac{C_{ab}}{\epsilon} = \frac{376.7}{\sqrt{\epsilon_r}}\,\frac{k\sqrt{G_aG_b}}{\sqrt{1 - k^2}}, \qquad (29)$$

where ϵ is the dielectric constant of the medium in which the waves propagate, ϵ_r is the relative dielectric constant of the medium, and 376.7 is the impedance of free space, calculated from $\sqrt{\mu_0/\epsilon_0}$.

By means of various available data [10]–[12], the normalized capacitances given in (27)–(29) can be related to physical dimensions of structures, thus permitting several possible realizations of the couplers to be made.

Broadbanding Matched Nonsymmetrical Directional Couplers

Matched nonsymmetrical directional couplers may be broadbanded, just as conventional directional couplers, by cascading two or more single-section couplers. Moreover, the existing tables of designs for asymmetrical and symmetrical multisection couplers [4]–[6] may be applied directly to designs for multisection nonsymmetrical directional couplers by interpreting the tables of designs in the proper way.

It is readily proved that nonsymmetrical directional couplers are mathematically equivalent to conventional directional couplers with transformers connected at both ports of one of the coupled lines. To show this, first replace B by A in (1) so that the matrix now represents a conventional directional coupler. Next, multiply rows 2 and 3 and columns 2 and 3 by $N=\sqrt{B/A}$. This operation returns the matrix to its original form with D replaced by ND. Since the preceding mathematical operation is physically equivalent to connecting transformers [14] of turns ratio $1:\sqrt{B/A}$ to Ports 2 and 3 of the conventional directional coupler, the equivalence $A=A'$, $N=\sqrt{B'/A'}$, and $D=D'/N$ is established, and the proof is complete. In the immediately preceding equations the primed symbols refer to the parameters of the nonsymmetrical directional coupler, and the unprimed symbols to those of the conventional directional coupler. Substitution of the primed and unprimed parameters into (13) shows that the coupling is invariant under these operations.

On the basis of this result, it can be concluded that the mathematical equivalence between stepped impedance lines and conventional directional couplers [15] carries over to nonsymmetrical directional couplers as well. By the same argument, the tables of designs for multisection conventional directional couplers [4]–[6] may be taken over for designs of multisection nonsymmetrical directional couplers.

For multisection nonsymmetrical directional couplers the design method is to cascade nonsymmetrical directional couplers which, when considered individually, have coupling coefficients corresponding to those in the tables. Because the data of Levy [4], Toulios and Todd [5], and Cristal and Young [6] are given in terms of normalized even-mode impedances, the coefficients of coupling may be obtained from the tables by the relationship

$$k = \frac{Z_{oe}^2 - 1}{Z_{oe}^2 + 1}\,. \tag{30}$$

In practice, the design procedure is as follows: 1) Choose a suitable design from one of the tables for multisection couplers; 2) calculate the coupling coefficient for each section by means of (30); and 3) obtain the even- and odd-mode admittances by (17)–(20), or the capacitances by means of (27)–(29). An example will be given to illustrate this procedure.

It is desired to design a -10-dB directional coupler to have less than 0.55-dB ripple and to have a 4 to 1 equal-ripple bandwidth. Let it also be assumed that an impedance transformation of 50 to 75 ohms is required. From Levy's design tables [4] it is found that a two-section asymmetrical directional coupler meets the requirements. The design obtained from the tables calls for one section to have $Z_{oe}^{(1)}/Z_0=1.5729$, and the second section to have $Z_{oe}^{(2)}/Z_0=1.1573$, where the superscript indicates the section number. The coefficient of coupling of each section is then computed from (30), giving

$$k^{(1)} = 0.4243 \tag{31}$$

$$k^{(2)} = 0.1450. \tag{32}$$

Finally, from (27)–(29), it is determined that the normalized capacitances for the two sections are

$$\text{Section 1}\quad C_a^{(1)}/\epsilon = 5.438/\sqrt{\epsilon_r},\quad C_b^{(1)}/\epsilon = 2.664/\sqrt{\epsilon_r},\quad C_{ab}^{(1)}/\epsilon = 2.882/\sqrt{\epsilon_r} \tag{33}$$

$$\text{Section 2}\quad C_a^{(2)}/\epsilon = 6.713/\sqrt{\epsilon_r},\quad C_b^{(2)}/\epsilon = 4.174/\sqrt{\epsilon_r},\quad C_{ab}^{(2)}/\epsilon = 0.9018/\sqrt{\epsilon_r}. \tag{34}$$

Mismatched Directional Couplers Having Infinite Directivity

The equations presented in previous sections showed that nonsymmetrical directional couplers may be mismatched and still have infinite directivity. It has been found that there are certain advantages to be obtained by purposely designing nonsymmetrical directional couplers to be mismatched: for a given coupling coefficient, both greater bandwidth and larger maximum impedance transformation ratio are obtained. These advantages must, of course, be weighed against the disadvantages of the mismatch. The exact relationships are presented in this section, along with coupling equations, design equations, and other data.

The condition for infinite directivity as given by (6) was

$$G_aG_b = AB - D^2.$$

If only this constraint is used, it is found that

$$\frac{v_4}{i_1} = \frac{G_b\sqrt{1+\tan^2\theta}}{G_aG_b + jAG_b\tan\theta} \tag{35}$$

$$\frac{v_2}{i_1} = \frac{jD\tan\theta}{G_aG_b + jAG_b\tan\theta}\,. \tag{36}$$

By applying the principle of conservation of energy, we obtain

$$|i_1|^2\,\mathrm{Re}\,(Z_{in}) = |v_4|^2G_b + |v_2|^2G_a. \tag{37}$$

Substituting (35) and (36) and solving for the Re (Z_{in}) gives

$$G_a\,\mathrm{Re}\,(Z_{in}) = \frac{(1-k^2)+\tan^2\theta}{(1-k^2)+R\tan^2\theta}, \tag{38}$$

where $R=AG_b/BG_a$ is the normalized input admittance at Port 1 at $\theta=90$ degrees.

From (38), we can readily reconstruct [13] $Z_{in}(s)$ as follows:

$$G_aZ_{in}(s) = \frac{\sqrt{1-k^2}+(1/\sqrt{R})s}{\sqrt{1-k^2}+\sqrt{R}\,s}\,. \tag{39}$$

Lastly, from (35), (36), and (39), the power coupling ratios and reflected power ratio are obtained:

$$\frac{P_2}{P_a} = \frac{k^2 \tan^2 \theta}{(1-k^2) + \frac{(R+1)^2}{4R}\tan^2\theta} \tag{40}$$

$$\frac{P_4}{P_a} = \frac{(1-k^2)(1+\tan^2\theta)}{(1-k^2) + \frac{(R+1)^2}{4R}\tan^2\theta} \tag{41}$$

$$\frac{P_r}{P_a} = \frac{\left(\frac{R-1}{2\sqrt{R}}\right)^2 \tan^2\theta}{(1-k^2) + \frac{(R+1)^2}{4R}\tan^2\theta}. \tag{42}$$

From (42) it is found that

$$r_{\max} = (\text{vswr})_{\max}$$
$$= R \quad \text{or} \quad \frac{1}{R} \text{ (whichever is greater than 1)} \tag{43}$$

and that $r_{\max}$ occurs at $\theta = 90$ degrees.

It is worth pointing out several conclusions that may be drawn from the previous expressions. First, dividing (36) by (35) gives

$$\frac{v_2}{v_4} = \frac{jD\sin\theta}{G_b}. \tag{44}$$

Thus v_2 and v_4 are 90 degrees out of phase for all frequencies.

Second, the ratio of P_2 to P_4 is

$$\frac{P_2}{P_4} = \frac{k^2}{1-k^2}\sin^2\theta, \tag{45}$$

which shows that the ratio of the power split between Ports 2 and 4 is independent of the amount of mismatch.

Third, from (39) it can be shown that

$$\left\{G_a Z_{in}(s)\right\}_{\text{Port 1}} = \left\{\frac{Y_{in}(s)}{G_b}\right\}_{\text{Port 2}}. \tag{46}$$

Thus the reflection coefficients at Ports 1 and 2 are negatives of each other.

Fourth, the ratio of P_2/P_a to $(P_2/P_a)_{\max}$ is given by

$$\frac{(P_2/P_a)}{(P_2/P_a)_{\max}} = \frac{\mathcal{R}u^2}{1+\mathcal{R}u^2} \tag{47}$$

where

$$u = \frac{\tan\theta}{\sqrt{1-k^2}} \tag{48}$$

$$\mathcal{R} = \frac{(r_{\max}+1)^2}{4r_{\max}}. \tag{49}$$

P_2/P_a will be 3 dB down from its maximum value when

$$\frac{\mathcal{R}u^2}{1+\mathcal{R}u^2} = \frac{1}{2}. \tag{50}$$

Solving (50) gives

$$u = \mathcal{R}^{-1/2}. \tag{51}$$

Therefore,

$$\theta_{3\,\text{dB}} = \tan^{-1}\left\{\sqrt{1-k^2}\,\frac{2\sqrt{r_{\max}}}{r_{\max}+1}\right\}, \tag{52}$$

where $\theta_{3\text{dB}}$ is the angle satisfying (50).

Note that for a given k, $\theta_{3\text{dB}}$ is a monotonic decreasing function of $r_{\max}$, and is a maximum for $r_{\max}=1$. Since the 3-dB bandwidths are given by formula

$$BW = \frac{180°}{\theta°_{3\,\text{dB}}} - 1, \tag{53}$$

it is readily demonstrated that the 3-dB bandwidths of nonsymmetrical directional couplers are always increased by designing the couplers to be mismatched. However, for weak coupling ($k^2 < 0.1$) and $r_{\max} < 2$, the increase in 3-dB bandwidth is less than 3.8 percent; and for larger values of k, the percentage increase in bandwidth is less. For $k = 0.707$ (-3.01-dB coupling for matched couplers) and $r_{\max} = 2$, the increase in bandwidth is only 2.9 percent. The fractional bandwidth[3] versus $r_{\max}$ for several values of coupling is shown in Fig. 3.

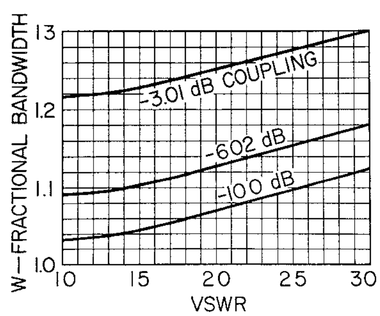

Fig. 3. Fractional bandwidth vs. maximum VSWR for nonsymmetrical directional couplers.

Design Equations for Mismatched Nonsymmetrical Directional Couplers

Recall that R as defined by (38) is the normalized input admittance at Port 1 when $\theta = 90°$, and is also the maximum VSWR or its reciprocal of the directional coupler. Therefore, in the design equations below, R

[3] The fractional-bandwidth w, is defined as

$$w = 2\left\{\frac{BW-1}{BW+1}\right\}.$$

should be chosen equal to the maximum VSWR (or its reciprocal) permitted by the design specifications. Either of the choices for the value of R will result in the same power relationships for the directional coupler. However, it generally will be easier to realize the coupler physically for one of the two choices. The design equations for mismatched nonsymmetrical directional couplers are as follows:

$$Y_{oe}{}^{a} = \frac{\sqrt{R}\, G_a - k\sqrt{G_a G_b}}{\sqrt{1-k^2}} \tag{54}$$

$$Y_{oo}{}^{a} = \frac{\sqrt{R}\, G_a + k\sqrt{G_a G_b}}{\sqrt{1-k^2}} \tag{55}$$

$$Y_{oe}{}^{b} = \frac{G_b/\sqrt{R} - k\sqrt{G_a G_b}}{\sqrt{1-k^2}} \tag{56}$$

$$Y_{oo}{}^{b} = \frac{G_b/\sqrt{R} + k\sqrt{G_a G_b}}{\sqrt{1-k^2}}. \tag{57}$$

For directional coupler geometries such as that shown in Fig. 2, the following equations apply:

$$\frac{C_a}{\epsilon} = \frac{376.7}{\sqrt{\epsilon_r}} \frac{\sqrt{R}\, G_a - k\sqrt{G_a G_b}}{\sqrt{1-k^2}} \tag{58}$$

$$\frac{C_b}{\epsilon} = \frac{376.7}{\sqrt{\epsilon_r}} \frac{G_b/\sqrt{R} - k\sqrt{G_a G_b}}{\sqrt{1-k^2}} \tag{59}$$

$$\frac{C_{ab}}{\epsilon} = \frac{376.7}{\sqrt{\epsilon_r}} \frac{k\sqrt{G_a G_b}}{\sqrt{1-k^2}}. \tag{60}$$

The theoretical limit on the amount of impedance transformation and coupling is now determined by (54) and (56). For $Y_{oe}{}^{a}$ and $Y_{oe}{}^{b}$ to be positive, it is necessary that

$$\frac{1}{k^2} \geq \frac{RG_a}{G_b} \text{ and } \frac{G_b}{RG_a}, \tag{61}$$

whichever is greater.

If R is chosen advantageously (61) may be stated in words: *Let the impedance (or admittance) transformation ratio of the directional coupler be taken as greater than one. Then, the reciprocal of the power coupling coefficient must be greater than or equal to the impedance (or admittance) transformation ratio divided by the maximum VSWR of the coupler.* Thus another result of designing nonsymmetrical directional couplers to be mismatched is that, for a given impedance transformation ratio, a larger coupling coefficient is theoretically possible; similarly, for a given coupling coefficient, a larger impedance transformation ratio is theoretically possible.

Experimental Results

To substantiate the analytical results for nonsymmetrical matched directional couplers, a trial −10-dB coupler having coupled lines of 50 and 75 ohms, respectively, was constructed and tested. From (27)–(29) it was determined that

$$C_1/\epsilon = 5.891 \tag{62}$$

$$C_2/\epsilon = 3.244 \tag{63}$$

$$C_{12}/\epsilon = 2.050, \tag{64}$$

where the subscripts 1 and 2 refer to the 50- and 75-ohm lines, respectively.

It was decided to construct the coupler using shielded coupled rectangular bars. Using the data of Getsinger [10], the following dimensions were determined.

$$t/b = 0.400 \text{ (chosen arbitrarily)} \tag{65}$$

$$(w/b)_1 = 0.508 \tag{66}$$

$$(w/b)_2 = 0.111 \tag{67}$$

$$s/b = 0.233. \tag{68}$$

Here b is the ground plane spacing, t is the thickness of the rectangular bars, w is the width of a coupled rectangular bar, and s is the spacing between coupled bars. The trial coupler was constructed with $\frac{5}{8}$-inch ground plane spacing in air dielectric. The nominal center frequency was 1.5 Gc/s. A drawing of the coupler, showing important dimensions, is given in Fig. 4, and a photograph of the constructed coupler is given in Fig. 5.

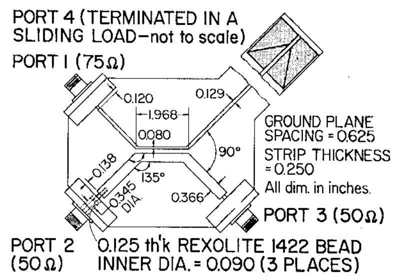

Fig. 4. Drawing of a trial nonsymmetrical −10-dB directional coupler with coupled lines of 50 and 75 ohms.

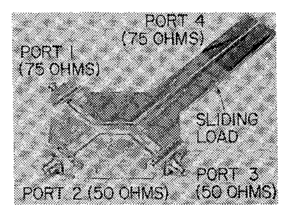

Fig. 5. Photograph of a trial nonsymmetrical −10-dB directional coupler with coupled lines of 50 and 75 ohms.

Type N connectors of 50 ohms were used at Ports 1, 2, and 3, while Port 4 was terminated internally in a sliding load. The 50-ohm connector at Port 1 (the 75-ohm port) was chosen for convenience. It was anticipated beforehand that the required measurements could be performed without being substantially affected by the mismatch.

The VSWR at Port 3 was measured with Port 2 terminated in a well-matched load. Since Port 1 is decoupled from Port 3, the discontinuity at Port 1 was not seen. The load at Port 4 had a sufficiently low VSWR when seen at Port 1 so that it also did not affect the measurements.

The VSWR at Port 1 was measured with Port 2 terminated in a well-matched load. The sliding load at Port 4 was adjusted to give first a maximum VSWR and next a minimum VSWR. During these measurements it was also noted whether the standing-wave minimum shifted 90 degrees or not. If there was a 90-degree phase shift, the VSWR at Port 1 was determined by

$$r = \sqrt{r_{\max}/r_{\min}}.$$

If there was no phase shift, the VSWR was determined by

$$r = \sqrt{r_{\max} r_{\min}}.$$

In Fig. 6 the results of these measurements are shown. The VSWR at Port 3 is seen to be quite low. It is less than 1.08 over most of the frequency band, 0.8 to 2.3 Gc/s. At Port 1, the VSWR varies about 1.5 (it should be exactly 1.5 in theory, as measured on a 50-ohm slotted line) without departing much from this value except at 0.8 and 0.9 Gc/s, where an unexplained resonance appears to occur. It should be pointed out that only a small effort was made to perfect the transition from the coaxial connector to the 75-ohm strip line, and that, undoubtedly, a portion of the VSWR at Port 1 is due to the transition itself. The power coupling from Port 2 to Port 1 was measured without compensating the discontinuity at Port 1. The VSWR of 1.5 at Port 1 corresponds to slightly less than 0.2-dB mismatch loss. The results of the measurements are given in Table I.

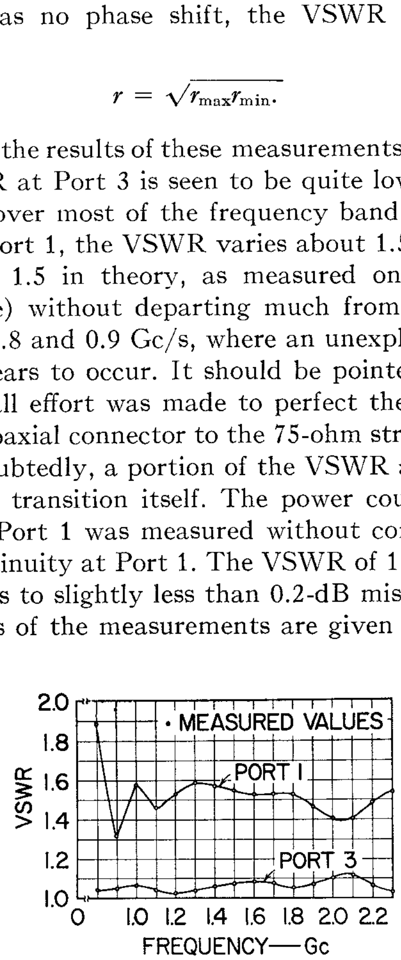

Fig. 6. VSWR at Ports 1 and 3 of trial nonsymmetrical −10-dB directional coupler. (The VSWR at Port 1 is with respect to a 50-ohm line.)

TABLE I

Power-Coupling From Port 2 to Port 1

Frequency (Gc/s)	Calculated (dB)	Measured (dB)
0.800	−12.4	−12.5
1.000	−11.1	−11.7
1.200	−10.4	−11.2
1.500	−10.0	−10.3
1.800	−10.4	−10.9
2.250	−12.8	−12.5
2.700	−19.8	−19.2

Originally it was intended to make directivity measurements using a double-sliding-load method. However, during the measurements it was found that the VSWR of the connector of the sliding load itself was too large to make the measurements meaningful. It was then decided to use a single sliding load (the load at Port 4) and to terminate Port 2 in a matched load.

Basically, the measurement procedure using a single sliding load is as follows. Power is made incident to Port 1. A reference level (denoted as R-dB) is established by measuring the power available at Port 2. Next, Port 2 is terminated in a matched load (by using a good load and a double stub tuner to give unity VSWR), and the power available at Port 3 is monitored. The sliding load at Port 4 is adjusted to give maximum power out of Port 3; let this value be Max-dB. Next, the sliding load is adjusted to give a minimum of power at Port 3; let this value be Min-dB. Also, let Δ equal the difference of Max-dB and Min-dB. Then it can be shown that the intrinsic directivity is as follows:

$$\text{Directivity} = 10 \log_{10} \left| \frac{S_{12}}{S_{13}} \right|^2$$

$$= (R - \text{Max}) - 10 \log_{10} \left\{ \frac{1 \pm 10^{-|\Delta/20|}}{2} \right\}^2 \text{dB}.$$

The ambiguity in the above formula can be resolved by making an appropriate phase measurement. However, this was not done in the present case; instead, the minimum value of the above formula was taken as the directivity.

Initially, directivity measurements were made without compensating the junctions of the coupled and uncoupled lines. The measured directivities at 1 and 1.5 Gc/s were approximately 18 dB. Next, compensating blocks similar to those used in the 3-dB coupler of Shimizu-Jones (see Fig. 5 of their paper) were located to maximize the directivity at 2.3 Gc/s and were fixed in that location for the remaining measurements. The results of the directivity measurements are given in Table II.

TABLE II

Directivity Measurements

Frequency (Gc/s)	Directivity (dB)
1.0	27
1.25	24
1.50	26
1.75	26.5
2.00	19
2.30	28.5

Conclusions

Nonsymmetrical directional couplers are equivalent to conventional directional couplers with impedance transformers at the two ports of one of the transmission lines. They may be designed to have infinite directivity and to be matched at all frequencies; or they may be designed to have infinite directivity at all frequencies, and a specified maximum mismatch. Mismatched nonsymmetrical directional couplers were shown to have greater bandwidth than matched nonsymmetrical directional couplers, although in most practical cases the increased bandwidth will be less than four percent. Both mismatched and matched nonsymmetrical directional couplers have theoretical limits on the amount of coupling and impedance transformation ratio that can be obtained simultaneously. The rule is that the reciprocal of the power coupling coefficient must be greater than or equal to the impedance transformation ratio, taken greater than one, divided by the maximum VSWR of the coupler. Matched nonsymmetrical directional couplers may be broadbanded by cascading additional nonsymmetrical couplers. Existing tables of designs for multisection asymmetrical and symmetrical couplers may be taken over directly by interpreting the design data as described in this paper.

Several possible applications for nonsymmetrical directional couplers are as directional detectors, power samplers, and reflectometers; they may be applied advantageously to the design of directional filters by permitting a change of impedance level of the resonant lines. A novel application is the construction of impedance transformers having dc isolation by properly cascading several nonsymmetrical directional couplers to make a 0-dB coupler.

Appendix I

Derivation of the Condition for Infinite Directivity

The Y-matrix of the nonsymmetrical coupled-transmission-line directional coupler was given in (1). For the following derivation let us write the defining equations as

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \tag{69}$$

where

$$\begin{aligned} y_{ij} &= \text{the } i,j \text{ entry of the matrix of (1)} \\ I_1 &= i_1 \\ I_2 &= \text{col}\,(i_2, i_3, i_4) \\ V_1 &= v_1 \\ V_2 &= \text{col}\,(v_2, v_3, v_4) \\ Y_{12} &= \text{row}\,(y_{12}, y_{13}, y_{14}) \\ Y_{12}^T &= \text{transpose of } Y_{12} \end{aligned}$$

$$Y_{22} = \begin{bmatrix} y_{22} & y_{23} & y_{24} \\ y_{23} & y_{22} & y_{12} \\ y_{13} & y_{12} & y_{11} \end{bmatrix}. \tag{70}$$

By terminating Ports 2, 3, and 4 in their respective loads, the following constraint is applied:

$$I_2 = -GV_2, \tag{71}$$

where

$$G = \begin{bmatrix} G_b & 0 & 0 \\ 0 & G_b & 0 \\ 0 & 0 & G_a \end{bmatrix}. \tag{72}$$

Equations (69) and (71) are easily combined, and upon the elimination of I_2 and V_1 yield

$$\frac{Y_{12}^T}{y_{11}} I_1 = \left\{ \frac{Y_{12}^T Y_{12}}{y_{11}} - (G + Y_{22}) \right\} V_2. \tag{73}$$

Solving for V_2 gives

$$V_2 = HY_{12}^T I_1, \tag{74}$$

where H is a 3×3 matrix having entries h_{ij} given by

$$H = \left\{ Y_{12}^T Y_{12} - y_{11}(G + Y_{22}) \right\}^{-1}. \tag{75}$$

The voltages at Ports 2, 3, and 4 are given by

$$\begin{bmatrix} v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} HY_{12}^T I_1. \tag{76}$$

Thus the directivity is given by

$$20 \log_{10} \left| \frac{v_2}{v_3} \right| = 20 \log_{10} \left| \frac{h_{11}y_{12} + h_{12}y_{13} + h_{13}y_{14}}{h_{12}y_{12} + h_{22}y_{13} + h_{23}y_{14}} \right|, \tag{77}$$

and the condition for infinite directivity is

$$h_{12}y_{12} + h_{22}y_{13} + h_{23}y_{14} \equiv 0. \tag{78}$$

Substituting in the values for h_{ij} and y_{ij} results in

$$AB - D^2 = G_a G_b. \tag{79}$$

Appendix II

Derivation of the Condition for an Impedance Match

The Y-matrix of the nonsymmetrical coupled-transmission-line directional coupler was given in (1). For the following derivation let us write the defining equations as

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & -Y_{11}(tC) \\ -(tC)Y_{11} & CY_{11}C \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}, \tag{80}$$

where

$$\left. \begin{aligned} I_1 &= \text{col}\,(i_1, i_2) \\ I_2 &= \text{col}\,(i_3, i_4) \\ V_1 &= \text{col}\,(v_1, v_2) \\ V_2 &= \text{col}\,(v_3, v_4) \\ t &= \sqrt{1 - s^2} \\ C &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ Y_{11} &= \frac{1}{s} \begin{pmatrix} A & -D \\ -D & B \end{pmatrix} \end{aligned} \right\}. \tag{81}$$

By terminating Ports 3 and 4 in their respective loads, the following constraint is applied:

$$I_2 = -(CGC)V_2, \tag{82}$$

where

$$G = \begin{pmatrix} G_a & 0 \\ 0 & G_b \end{pmatrix}. \tag{83}$$

Equations (82) and (80) may be combined, and upon elimination of I_2 and V_2, yield

$$I_1 = Y_{11}\{\mathcal{I}_2 - t^2(G + Y_{11})^{-1}Y_{11}\}V_1, \tag{84}$$

where $\mathcal{I}_2$ is a 2×2 identity matrix. Equation (84) may be simplified by noting that

$$-(G + Y_{11})^{-1} = \frac{T(G + Y_{11})T}{|(G + Y_{11})|} \tag{85}$$

where

$$T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{86}$$

and $|(G + Y_{11})|$ is the determinant of the matrix $G + Y_{11}$. Thus,

$$I_1 = Y_{11}\left\{\mathcal{I}_2 + t^2 \frac{T(G + Y_{11})TY_{11}}{|G + Y_{11}|}\right\} V_1. \tag{87}$$

Let the entries of an M matrix be M_{ij}, and let

$$M = Y_{11}\left\{\mathcal{I}_2 + t^2 \frac{T(G + Y_{11})TY_{11}}{|G + Y_{11}|}\right\}. \tag{88}$$

Then

$$\frac{i_1}{v_1} = (Y_{in})_a = \frac{\begin{vmatrix} M_{11} & M_{12} \\ M_{12} & G_b + M_{22} \end{vmatrix}}{G_b + M_{22}} = M_{11} - \frac{M_{12}{}^2}{G_b + M_{22}}. \tag{89}$$

The condition for a match at Port 2 is

$$G_a - Y_{in} = 0, \tag{90}$$

which when combined with (89) yields the following determining equation:

$$\text{Numerator of } \{G_aG_b + G_aM_{22} - G_bM_{11} - (M_{11}M_{22} - M_{12}{}^2)\} \equiv 0. \tag{91}$$

Substituting the appropriate values for M_{ij} into (91), and equating coefficients of the variables s_i equal to zero, yields

$$\left.\begin{aligned} \frac{G_a}{G_b} &= \frac{A}{B} \\ G_aG_b &= AB - D^2 \end{aligned}\right\}. \tag{92}$$

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